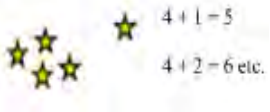
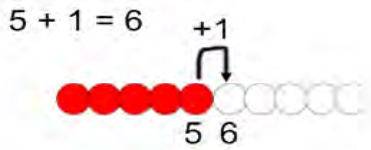
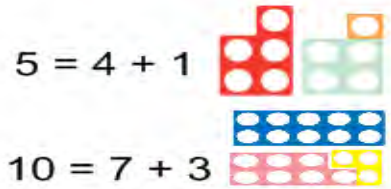
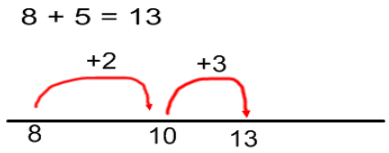
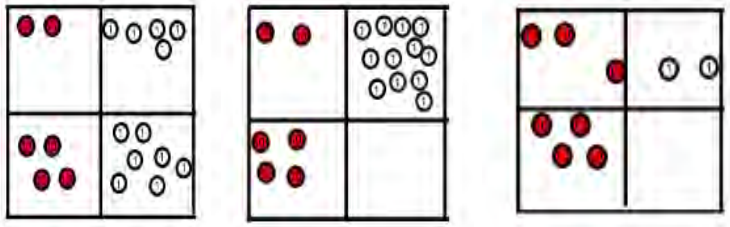
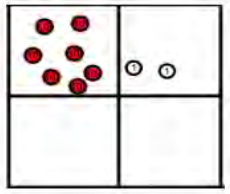
+ Addition +

We encourage children to use mental methods whenever possible
Children may not need to be taught every step

STEP	Tips for mental / oral session	Concept & images	Comments
1 *Early addition*	Counting	Combining groups of objects to find the total  Then adding on to a set, one by one  	Put all objects together and count… Find total of 2 groups using objects Then total of 2 groups using objects and numerals Then… total of 2 groups using objects and recording as a number sentence… EARLY MATHS CHALLENGES SUPPORT THIS STEP **Requires fluency with counting from any number!** **Use fingers (but avoid counting from one each time!)** **NUMICON very useful here**
2 *Relating groups of objects to number line*	Finding numbers	'Informal number line' / number sentences  As above, alongside a calculation  	Look at number sentences. Then… Look at number sentences – use objects provided to find the answer Look at number sentences: what do we have to do? Use objects to find an answer
3 ***Locating numbers on a number line / track & adding one more.***	Counting to ten and back Locating numbers	Add one onto a number  	Find 5 on number track, then add one Encourage children to locate the first number and count on from there, rather than starting at zero.

4 *Number bonds **up to** 10* *i.e. number bonds for all numbers up to 10, not just 10 itself*	Pegs on a coat hanger (turn round to show inverse) Write all the ways to make 2,3,4,5,6,7,8,9,10	How many ways of splitting up a number? 10 = ? + ? 9 = ? + ? 8 = ? + ? Etc 	Model with numicon In order to calculate effectively children must know all the bonds for numbers up to ten. This will enable them to **jump** on the number line rather than *count*. Children need to understand = as balance Using a bead bar is also an effective way to showing how to split smaller numbers up MATHS CHALLENGES SUPPORT THIS STEP
5 *Using number bonds to add on the number line.*	Adding to a ten mentally (10 + 2 = 12, 10 + 3 = 13 10 + 4 = 14…)	Bridge 10 (e.g. 8 + 5 =13) Include use of bead bar  Add a 2digit number and a 1 digit number	Emphasise JUMP on number line, NOT counting! Use number bonds to jump to the next ten on the number line. Then add what is left in one jump.4 THIS IS AN IMPORTANT STEP AND IT OFTEN REQUIRES A LOT OF PRACTISE
6 TU + TU *Working towards the compact standard method (so child can choose)*	Counting on in tens *Focus on tricky parts: counting over 100, counting back past 20 in the teen numbers.*	TU + TU Use bundles of straws first as these help children to develop an understanding of the size of a number then Use diennes first and finally move on to place value counters.  2 5 + 4 7 2 1 Move all the ones together and EXCHANGE ten ones for a ten counter	**Children should be working on these methods during Year2/3**

		2 5 + 4 7 7 2 1 	
7 *Compact Column addition HTU+HTU*	Practice partitioning Number bonds **up to** ten *(to avoid counting in ones when adding up columns)*	347 + 122 347 Then 347 Then, with carrying 159 +122 +122 +264 9 469 423 60 1 400 469 AGAIN, use PLACE VALUE COUNTERS to model CARRYING if needed	As the children become more confident in column addition they can gradually start to use the compact method for speed. **It is vital that they still understand that the small '1' represents tens or hundreds (if they don't, use place value counters)**
8 *Compact Column addition with decimals*	'Moving digits' ITP to investigate decimals. Practice partitioning Number bonds **up to** ten *(to avoid counting in ones when adding up columns)*	Same number of decimal places 78.5 km +54.6 km 133.1 km 1 1 Then, different number of decimal places 124.9 + 7.25 132.15 1 1	As with the compact column addition strategy it is vital that children understand what each column represents in terms of value.

-Subtraction -

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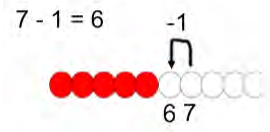
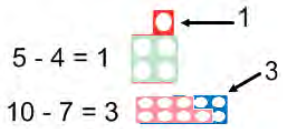
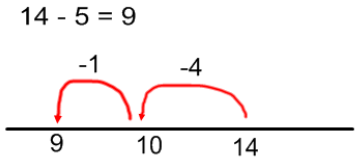
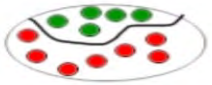
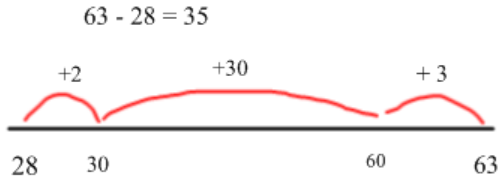
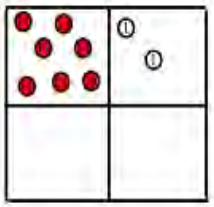
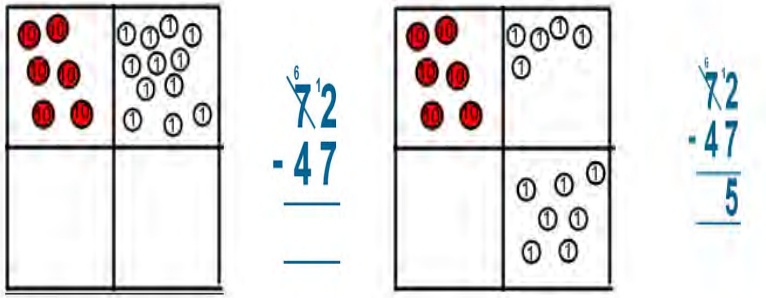
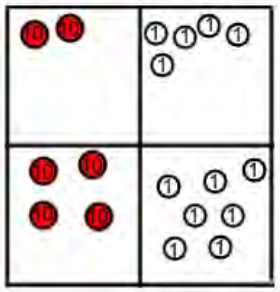
STEP	Tips for mental / oral session	Concept & images	Comments
1 Early subtraction	Counting	Take away a number of objects from the group, count what's left Introduce – and = symbols 	Then… start with group of objects and record the numeral. Take some away, record and count what's left (record) '6 take away 3 is 3 OR 3 less than 6 is 3'
2 Relating groups of objects to number lines	Finding numbers	Include vocabulary: 'difference' Relate to number line 	Emphasise JUMPING on number line, not counting Then… look at a number line: what do we need to do?
3 Locating numbers on a number line and finding one less.	Counting to ten and back Locating numbers	Take away one from a number 	Find 5 on number track, then SUBTRACT one Encourage children to locate the first number and count back from there, rather than starting at zero.
4 Number bonds **up to** 10.	Use pegs on washing line (reverse to show inverse)	Inverse use of number bonds(the opposite of step 3 for addition) 	Model with numicon In order to calculate effectively children must know all the bonds for numbers up to ten. This will enable them to **jump** back on the number line rather than *count*. Using a bead bar is also an effective way to showing how to split smaller numbers up. **KS1 children to also model this using jumps on a number line in order to lead to step 5.**
5 Using number bonds to jump back on the number line / begin to find	Place value partitioning Children need to know number bonds to 20 AND to all the	Jumping back (Bridging 10) 	Emphasise JUMP on number line, NOT counting! Use number bonds to jump back to the previous ten on the number line. Then subtract what is left in one jump. Use number bonds/love hearts

differences by comparing	numbers up to 20… N.B. BUNDLES of 10 STRAWS ARE BETTER THAN DIENNES TENS in aiding understanding	**Comparing two sets (comparison or difference**  **12 -5 =7** **Seeing one set as partitioned**  Seeing 12 as 5 and 7	**Useful when two numbers are 'close together', where 'take-away' image can be cumbersome** **Helps to see the related calculations; 5+7=12, 7+5=12, 12-7 = 5 and 12-5=7 as all in the same diagram**
6 TU - TU Subtracting on the number line (finding the difference)	Counting on and back in tens *Focus on tricky parts: counting over 100, counting back past 20 in the teen numbers.*	TU – TU start with multiples of 10 i.e. 60-40, count up from 40 COUNTING ON (on a numberline)  	Emphasise looking at HOW CLOSE NUMBERS ARE before diving into use of a number line. Can this be done mentally by counting on from the lower number? THE NUMBER LINE REPRESENTS THE JUMPS IN YOUR HEAD, SO ADD THE JUMPS! *If subtracting near multiples of ten, more confident pupils can do subtracting a ten and adjusting:* *43 – 19 can be done by 43 - 20 = 23* *Add one back on = 24* *MATHS CHALLENGES SUPPORT THIS STEP*
7 Column subtraction without exchange	Practice partitioning Number bonds **up to** ten *(to avoid counting in ones when subtracting columns)*	Easy column subtraction to practise layout. USE DIENNES AND THEN PLACE VALUE COUNTERS TO SUPPORT THIS STEP 73 Then 567 - 41 - 342 32 225	Don't use number line for HTU – HTU (only exception is something like 1,000 – 279, which would involve too many exchanges)
8 Column subtraction with exchange	Practice partitioning Number bonds **up to** ten *(to avoid counting in ones subtracting columns)*	Compact column subtraction with EXCHANGE(S) 2 1 Then… 1 4 2 1 1 ~~3~~ 7 ~~5~~ ~~3~~ 6 - 2 9 - 2 7 7 1 0 8 2 5 9 If child struggles, use PLACE VALUE COUNTERS to SHOW THE EXCHANGING e.g. 72 – 47	As the children become more confident in column subtraction they can gradually start to use the compact method for speed. **It is vital that they still understand that the small '1' represents tens or hundreds ~ if they don't use PLACE VALUE COUNTERS**

		 Start off with 7 tens and 2 ones. EXCHANGE one of the tens for ten ones  	Note how the answer is clearly shown on the top row (it's literally what you're left with. The amount taken away is on the bottom: helps with checking).
9 Compact column subtraction with decimals.	'Moving digits' ITP to investigate decimals. Practice partitioning Number bonds **up to** ten *(to avoid counting in ones subtracting columns)*	1 6 1 1 ~~7~~ ~~2~~ . 5 km - 4 . 6 km 6 7 . 9 km Then, subtractions with different numbers of decimal places	As with the compact column subtraction strategy it is vital that children understand what each column represents in terms of value.

X Multiplication X

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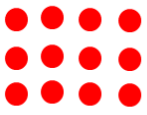
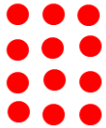
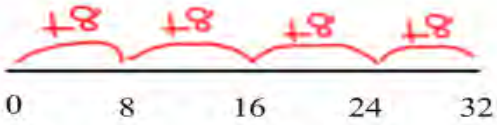
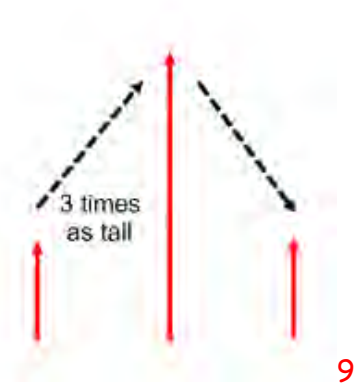
Children may not need to be taught every step *– they may be comfortable with using standard method if their times tables knowledge / understanding of place value is secure*

STEP	Tips for mental / oral session	Concept & images	Comments
1 Repeated addition	Counting in steps of… Times tables songs Counting forwards and backwards	5 x 3 = 15 is the same as 5 + 5 + 5 = 15  "5" "10" "15" 	Repeatedly adding the same number again and again. Counters can be used to illustrate OR BEAD BARS Good to say 'How many 5s are there in 15?', not just 5 x 3 =15
2 Simple Multiplication	Counting in steps of… Times tables songs Times Tables bingo Interactive maths games.	Array 3 x 4 = 12  Number 3 6 9 12 line 4 x 3 = 12  4 8 12 8 x 4 = 32  0 8 16 24 32	Read out the calculations as: 3x4 '3, multiplied 4 times' Understand that this is a group of 3, repeated 4 times. Use an array to model the concept. Emphasise that children don't count individual dots, but count up rows / columns This can lead onto children representing their counting on a number line.
3 Doubling (to help learn times tables) SCALING	Interactive maths games. Doubling & Halving rapid recall	To do 8 x 8… 8 x 2 = 16 *(double or x2)* 8 x 4 = 32 *(double, then double again or x4)* *8 x 8 = 64 (double once more, x8)*	Use language of 'twice as big…'

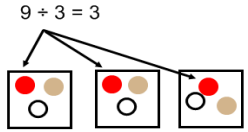
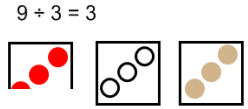
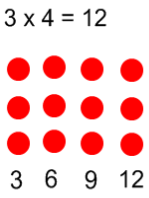
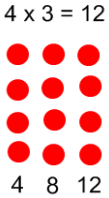
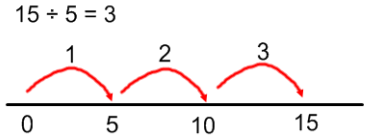
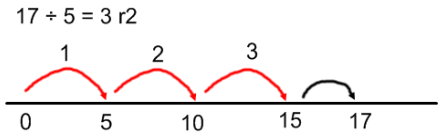
		 9 x 1/3 = 3 OR 3 is 1/3 of 9 '3 times shorter'	X means SCALE UP OR DOWN Images like this will help younger children to understand fractions better (and fraction walls will too)
4 Moving digits	Moving digits ITP	PLACE VALUE CHARTS are valuable for whole numbers and decimals 1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100 200 300 400 500 600 700 800 900 1000 2000 3000 4000 5000 6000 7000 8000 9000 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100 200 300 400 500 600 700 800 900 Multiply by 10 / 100 etc. 7.9 x 100 = 790 H T U . tenths 7 . 9 7 9 x 10 (digits move one column to left) 7 9 0 x 100 (digits move two columns to left)	Similar to doubling, children should be able to multiply by ten mentally. They need to do this in order to solve larger multiplication calculations effectively. Use MOVING DIGITS ITP / place value chart: Emphasise the DIGITS MOVE… Saying 'add on a zero when x by 10 / two zeros when x by 100' etc. is fine so long as it's emphasised that this ONLY WORKS FOR WHOLE NUMBERS…. Or children will put £1.75 x 10 = £1.750!!!)

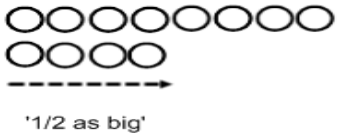
5 Grid Method TU x U & Working towards Standard Method	Times Tables bingo Interactive maths games.	Partitioning works well as a mental method, but children often make mistakes when using it as a written method, so avoid it (especially with TU x TU e.g. 14 x 12 they will put 108, because they think '10 tens are one hundred, 4 x 2 is 8, add that together. They need to think what **TEN TIMES 14** is first…) '13 x 6 is the same as 10 x 6 and 3 x 6 (60 + 18) = 78' is fine. … So use Grid Method if not absolutely sure of the answer x 10 3 60 + 18 = 78 6 60 | 18 Work towards standard method 2 4 X 3 1 2 (x3) 6 0 (x20) 7 2	The grid method allows children to use known number facts to solve multiplication problems.
6 Compact Standard Method	Times Tables bingo Interactive maths games.	Compact (standard method) 2 4 X 3 7 2 1	Children can begin to use the compact methods when they are comfortable with solving the multiplications mentally.
7 Grid Method (Or standard method if children are confident) for TU X TU	Times Tables bingo Interactive maths games.	24 x 14 = 336 X 20 4 10 | 200 | 40 4 | 80 | 16 200 80 40 + 16 336	The grid method allows children to break up large multiplication calculations into easier calculations. *Being able to multiply digits by ten and multiples of ten is a necessity.*

Grid Method (Or standard method if children are confident) for HTU X U		24 X 14 96 (4 x 4 = 16, plus 4 x 20 = 80) 240 (10 x 24= 240) 336 Then 242 x 7 = 1694 X 200 40 2 7 1400 | 280 | 14 1400 280 + 14 1694 242 X 7 1694 2 1	
8 Multiplying Decimals	Times Tables bingo Interactive maths games.	14.53 x 4 = 58.12 14.53 X 4 58.12 1 2 1	N.B. Grid Method is best avoided with decimals, because identifying the value of each digit beyond the decimal point is tricky for children Continue to use 0 as place holder if necessary .

÷ Division ÷

Remember, children should think about whether a calculation can be done mentally first.

STEP	Tips for mental / oral session	Concept & images	Comments
1 Sharing	Counting in groups.	SHARING 'Is it fair?' 	USE COUNTERS OF DIFFERENT COLOURS When sharing you know how many groups you will have; you are working out how many will be in each group. **Don't 'over – teach' sharing!** **Focus more on grouping**
2 Grouping (and use of arrays)	Times tables facts. Times table bingo	As GROUPING – link to REPEATED ADDITION/x tables 9 ÷3 = 3 (groups)  Arrays are useful to explain RELATED FACTS: 3 x 4 = 12 4 x 3 = 12 So 12 ÷ 4 = 3   And 12 ÷ 3 = 4	As this relies more on times tables knowledge, it is better to use this strategy than sharing. Children should understand that even when solving a 'sharing' problem, they can solve it quicker through grouping. MULTIPLICATION AWARDS SUPPORT THIS WORK
3 Grouping on the number line Then with remainders	Times tables facts. Times table bingo Primary resources interactive games	Grouping using number line 15 ÷ 5 = 3  Finding a remainder 17 ÷ 5 = 3 r 2 	Encourage children to read the question as: *'I have 15, how many 5s?'* They can then use times tables knowledge to solve the problem, using number lines to record their thinking. Encourage children to read the question as: 'I have 17, how many 5s?'

			How many WHOLE groups of 5 can they count in 17? What's left over? *This is the remainder.*
4 Begin to link to SCALING	HALVING (use Numicon / Bead Bars / counters)	**8 ÷ 2 is the same as 8 x ½** (or 8/2) = 4 SCALING is linked with SHARING:  '1/2 as big' **LINK WITH DIVISION e.g. show 9 ÷ 3 is 9 x 1/3** **Also show 12 ÷ 4 as 12 so that they are introduced to improper fractions.** **4**	**The new 2014 Curriculum states that children should be introduced to fractions at an early age (around Year 2)** **This introduction to improper fractions with simplifying when they get older, e.g. 120 ÷ 15 = 120 24** **15 = 3 = 8**
5 TU÷U Standard method (move onto HTU÷TU too) Then HTU÷U	Times tables rapid recall	Standard 'Goes Into' Method 1 4 r 2 5 ⟌ 7 2 2 Then 1 6 4 r 3 6 ⟌ 9 3 8 2 7 = 164 3/6	Using the standard 'goes into' method allows children to use known multiplication facts mentally and reduce the jottings needed to record their thoughts.
6 Decimal **AND FRACTION** Divisions	Decimal place value Times tables rapid recall	With decimals: Use standard method 1 2 r 3 . 5 87.5 ÷ 7 = 7 ⟌ 8 1 7 3 . 5 3/5 ÷ 2/3 = 3/5 X 3/2 = 9/10 **'Flip' the 2nd fraction, then multiply**	More able could put answer to calculation on left as 12.5, because they'd notice that 3.5 is half of 7